Exercise 1

$$p(\theta) = \frac{1}{2} \qquad 0 < \theta < 2$$

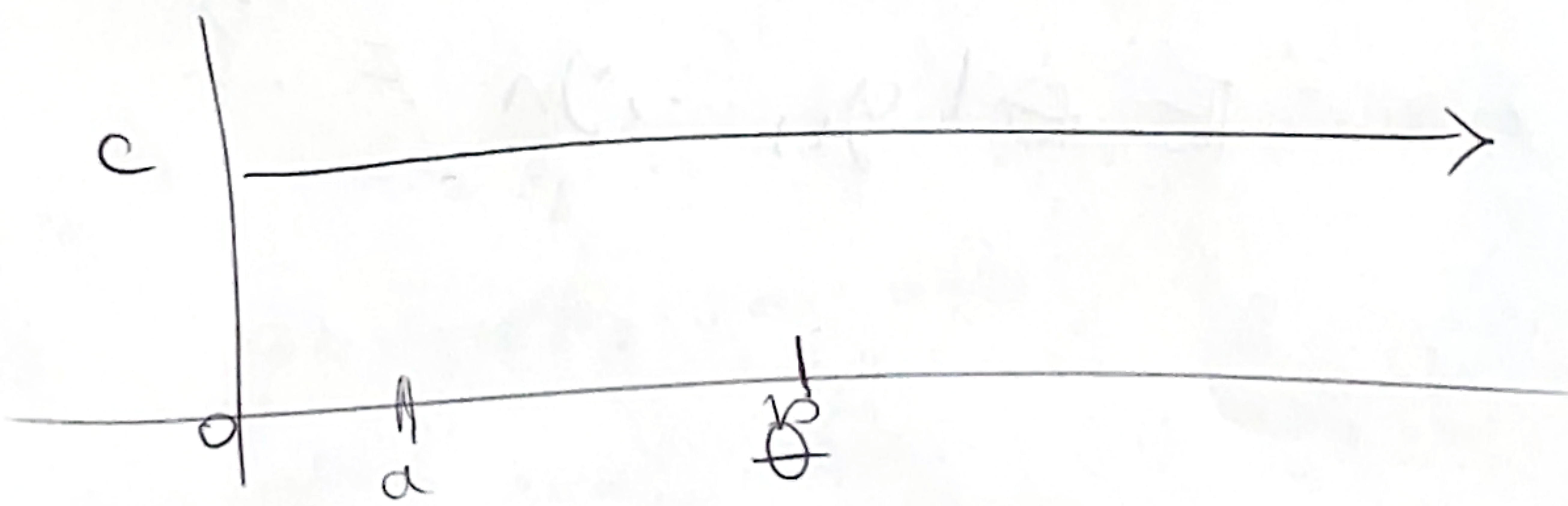
$$\phi = \log\theta \quad \Rightarrow \quad e^{\phi} = \theta$$

$$p(\phi)\, d\phi = p(\theta)\, d\theta$$

$$p(\phi) = \underbrace{p(\theta)\left|\frac{d\theta}{d\phi}\right|}_{\frac{1}{2}\, e^{\phi}} \qquad -\infty < \phi < \log 2$$

* A prior on θ induces a prior on $\phi = g(\theta)$.

Improper Uniform example



$$p(\theta) \propto c \qquad \theta > 0$$

$$0 < v < 1 \qquad v = l^3$$

$l \sim \text{Unif}(0,1)$

$l^3 \sim \text{Unif}(0,1)$

Exercise 2

We want $Pr(\tilde{y}_1 < \tilde{y}_2 \mid \vec{y}_1, \vec{y}_2)$

$\tilde{y}_1$: new obs. from group 1

$\tilde{y}_2$: new obs. from group 2

To compute w/ Monte Carlo sampling we need:

$$p(\tilde{y}_1 \mid \vec{y}_1, \vec{y}_2) \text{ and}$$
$$p(\tilde{y}_2 \mid \vec{y}_1, \vec{y}_2)$$

By the independence assumed in our model:

$$p(\tilde{y}_1 \mid \vec{y}_1, \vec{y}_2) = p(\tilde{y}_1 \mid \vec{y}_1)$$
$$p(\tilde{y}_2 \mid \vec{y}_1, \vec{y}_2) = p(\tilde{y}_2 \mid \vec{y}_2)$$

In general, when $\tilde{y}$ is a new observation from the same pop'n, we call $p(\tilde{y} \mid y_1, \dots, y_n)$ the "posterior predictive distribution"!!

$$p(\tilde{y} \mid y_1, \dots, y_n) = \int p(\tilde{y} \mid \theta)\, p(\theta \mid y_1, \dots, y_n)\, d\theta$$

by rule of marginal prob.

$$\begin{matrix} y_1 & \searrow & & & \\ \vdots & \rightarrow & \theta & \rightarrow & \tilde{y} \\ y_n & \nearrow & & & \end{matrix}$$

$$p(\tilde{y} \mid \vec{y}) = \int p(\tilde{y} \mid \theta)\, p(\theta \mid \vec{y})\, d\theta$$

We know how to simulate from $p(\tilde{y} \mid \theta)$ (our data generative model) & we know how to simulate from $p(\theta \mid \vec{y})$.

So we can proceed w/ Monte Carlo integration (approximation)

To approximate the integral above:

1. Sample $\theta^{(s)} \sim p(\theta \mid \vec{y})$
2. Sample $\tilde{y}^{(s)} \sim p(\tilde{y} \mid \theta^{(s)})$