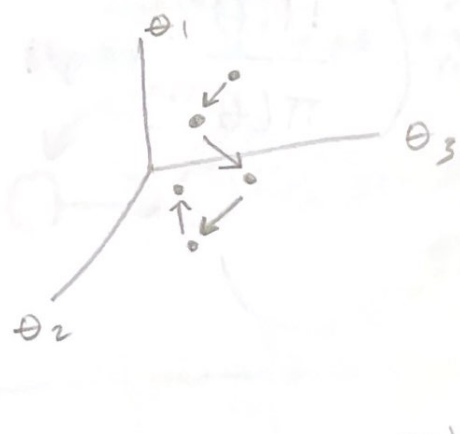
Recap:

- We want samples from $p(\theta_1, \ldots, \theta_n \mid \vec{y})$.

- We view "parameter space" $\to$ "state space" as a physical space: e.g let $n=3$:

- We view <u>MCMC</u> as a particle moving through parameter space.



- The "history" histogram of where the particle has been in θ_i approximates $p(\theta_i \mid \vec{y})$.

Recap: <u>Metropolis-Hastings</u> (LAB).

Ex 1 Let θ be a r.v. w/ support over the integers

$$\theta \in \{-\infty, \ldots, -2, -1, 0, 1, 2, \ldots, \infty\}$$

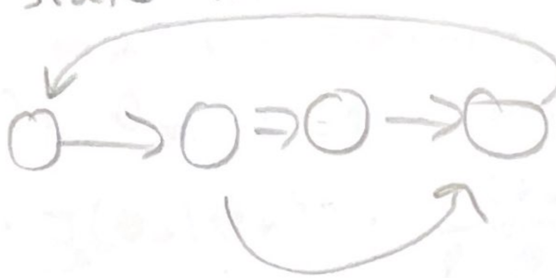
Let $\theta^{(0)} = 0$.

Let $J(\theta \mid \theta^{(s)}) = \theta^{(s)} \pm 2$

Our Markov chain is reducible.

Ex 2.1 Imagine we cannot stay in the same state in the ex. above.

Periodic



← Ex 2.2

Ex/ Imagine discrete $\theta \in \begin{cases} A \\ B \\ C \end{cases}$

not recurrent

If we do not continue sampling state A,

$$Pr(\theta^{(s)} \in A) \to 0 \quad \text{as } s \to \infty$$

$\pi(\theta)$:= stationary distr.
$p_0(\theta)$:= posterior distr. "target" of MH MCMC

Let θ discrete r.v.

T.P. $\pi(\theta) = p_0(\theta)$

put another way
I want t.p. that if $pr(\theta^{(s)} = \theta) = p_0(\theta)$
then $pr(\theta^{(s+1)} = \theta) = p_0(\theta)$

because if p_0 is a stationary distr.
it is the sationary distr. by uniqueness.

Let θ_a & θ_b be two values of θ
s.t.
$p_0(\theta_a) \cdot J_s(\theta_b | \theta_a) \geq p_0(\theta_b) J_s(\theta_a | \theta_b)$
Then under MH the probability $\theta^{(s)} = \theta_a$
and $\theta^{(s+1)} = \theta_b$ is given by

$$\underbrace{\cancel{p_0(\theta_a)}}_{\text{prob ending up in state } \theta_a} \cdot \underbrace{\cancel{J_s(\theta_b | \theta_a)}}_{\text{prob proposing } \theta_b \text{ from } \theta_a} \cdot \underbrace{\frac{p_0(\theta_b) J_s(\theta_a | \theta_b)}{\cancel{p_0(\theta_a) J_s(\theta_b | \theta_a)}}}_{\text{prob accepting}}$$

$$= \boxed{p_0(\theta_b) J_s(\theta_a | \theta_b)}$$

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other way: $pr(\theta^{(s)} = \theta_b, \theta^{(s+1)} = \theta_a)$

$$\boxed{p_0(\theta_b) \cdot J_s(\theta_a | \theta_b) \cdot 1}$$

★ So $\forall \theta_a, \theta_b$

$$\Pr(\theta^{(s)} = \theta_a, \theta^{(s+1)} = \theta_b) = \Pr(\theta^{(s)} = \theta_b, \theta^{(s+1)} = \theta_a)$$

T.D. $\Pr(\theta^{(s+1)} = \theta) = \boxed{p_0(\theta)}$

$$= \Pr(\theta^{(s)} = \theta)$$

Proof

$$\Pr(\theta^{(s+1)} = \theta) = \sum_{\theta_A} \Pr(\theta^{(s+1)} = \theta, \theta^{(s)} = \theta_A)$$

by ★

$$= \sum_{\theta_A} \Pr(\theta^{(s+1)} = \theta_A, \theta^{(s)} = \theta)$$

$$= \Pr(\theta^{(s)} = \theta) \quad ■$$