1. Let y_i be # of assists made by a particular player in a particular game.

- Support of y? $y \in \{0, 1, 2, 3, 4, \ldots\}$
- $y_i | \lambda \sim \text{Poisson}(\lambda)$

Let $\lambda = \theta x$

x : # minutes a particular plays in a particular game.

θ : rate of assists per unit time

$$p(y_1, \ldots, y_n | \theta) = \prod_{i=1}^{n} p(y_i | \theta)$$

$$= \prod_{i=1}^{n} \frac{(\theta x_i)^{y_i} e^{-\theta x_i}}{y_i!}$$

2. Prior of unknowns

- What is unknown? answer: θ
- What is its support? answer: $\theta > 0$

$\theta \sim \text{gamma}(a, b)$

Choose $a = 9$, $b = 3$ s.t. $E(\theta) = 9/3 = 3$.

^ terrible prior, can our likelihood overcome it?

3. Posterior

$$p(\theta \mid y_1, \dots, y_n) \propto \underbrace{p(y_1, \dots, y_n \mid \theta)}_{\text{likelihood}} \overbrace{p(\theta)}^{\text{prior}}$$

$$\propto \left(\theta^{\sum y_i} e^{-\theta \sum x_i}\right) \cdot \left(\theta^{a-1} e^{-b\theta}\right)$$

$$\propto \underbrace{\theta^{\sum y_i + a - 1} e^{-\theta(b + \sum x_i)}}$$

kernel of gamma(α, β)

$\alpha = \sum y_i + a$

$\beta = b + \sum x_i$

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4. What is $E(\theta \mid y_1, \dots, y_n)$?
"posterior expectation of θ"

$$\frac{\alpha}{\beta} = \frac{\sum y_i + a}{b + \sum x_i}$$

How is this diff't from prior expectation?

$E\theta = a/b$

$\sum y_i$ = total assists

$\sum x_i$ = total min / played

prior says a assists in b minutes.

$\text{var}(\theta \mid y_1, \dots, y_n) = \alpha/\beta^2$

Poisson as a member of the exp. family

$$p(y|\lambda) = \underbrace{\frac{1}{y!}}_{h(y)} \lambda^y e^{-\lambda}$$

Now λ^y needs to look like $e^{\phi t(y)}$

Let $\boxed{\phi = \log \lambda}$, then $e^{\phi} = \lambda$

$$p(y|\phi) = \boxed{\frac{1}{y!}} \; \boxed{e^{\phi y}} \; \boxed{e^{-e^{\phi}}}$$

$\frac{1}{y!} \rightarrow h(y)$; $e^{\phi y} \rightarrow e^{\phi t(y)}$; $e^{-e^{\phi}} \rightarrow c(\phi)$

$t(y) = y$

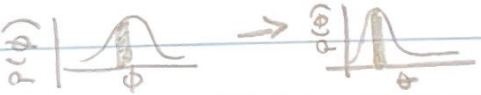
therefore the conj. prior is

$$p(\phi|n_0, t_0) = c(\phi)^{n_0} e^{n_0 t_0 \phi}$$

$$= e^{-e^{\phi} \cdot n_0} e^{n_0 t_0 \phi}$$

how to go from $p(\phi|n_0, t_0) \rightarrow p(\lambda|n_0, t_0)$?

one-line formula:



typo: plot on right should be "lambda" not "theta"

$$p(\phi|n_0, t_0)\, d\phi = p(\lambda|n_0, t_0)\, d\lambda$$

$$p(\lambda|n_0, t_0) = p(\phi|n_0, t_0) \left| \frac{d\phi}{d\lambda} \right| \qquad \frac{d}{d\lambda} \log \lambda = \frac{1}{\lambda}$$

$$\underbrace{e^{-e^{\log\lambda}} \cdot n_0 \, e^{n_0 t_0 \log\lambda}} \cdot \left| \frac{d\phi}{d\lambda} \right|$$

$$= e^{-\lambda n_0} \lambda^{n_0 t_0} \cdot \left| \frac{1}{\lambda} \right|$$

$$= \underbrace{e^{-\lambda n_0} \lambda^{n_0 t_0 - 1}}$$

$$\text{gamma}(n_0 t_0, n_0)$$